4.2 Apply Congruence and Triangles

Before You identified congruent angles.

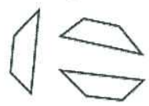
Now You will identify congruent figures.

Why? So you can determine if shapes are identical, as in Example 3.

Key Vocabulary
- **congruent figures**
- **corresponding parts**

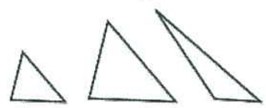
Two geometric figures are *congruent* if they have exactly the same size and shape. Imagine cutting out one of the congruent figures. You could then position the cut-out figure so that it fits perfectly onto the other figure.

Congruent



Same size and shape

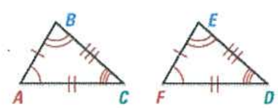
Not congruent



Different sizes or shapes

In two **congruent figures**, all the parts of one figure are congruent to the **corresponding parts** of the other figure. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

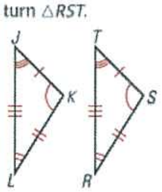
CONGRUENCE STATEMENTS When you write a congruence statement for two polygons, always list the corresponding vertices in the same order. You can write congruence statements in more than one way. Two possible congruence statements for the triangles at the right are $\triangle ABC \cong \triangle FED$ or $\triangle BCA \cong \triangle EDF$.



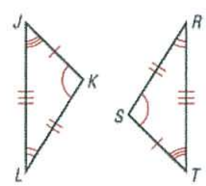
Corresponding angles	$\angle A \cong \angle F$	$\angle B \cong \angle E$	$\angle C \cong \angle D$
Corresponding sides	$\overline{AB} \cong \overline{FE}$	$\overline{BC} \cong \overline{ED}$	$\overline{AC} \cong \overline{FD}$

EXAMPLE 1 Identify congruent parts

VISUAL REASONING
To help you identify corresponding parts, turn $\triangle RST$.



Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



Solution

The diagram indicates that $\triangle JKL \cong \triangle TSR$.

Corresponding angles $\angle J \cong \angle T$, $\angle K \cong \angle S$, $\angle L \cong \angle R$

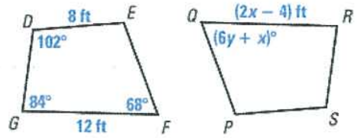
Corresponding sides $\overline{JK} \cong \overline{TS}$, $\overline{KL} \cong \overline{SR}$, $\overline{LJ} \cong \overline{RT}$

EXAMPLE 2 Use properties of congruent figures

In the diagram, *DEFG* ≅ *SPQR*.

a. Find the value of x.

b. Find the value of y.



Solution

a. You know that $\overline{FG} \cong \overline{QR}$.

$FG = QR$

$12 = 2x - 4$

$16 = 2x$

$8 = x$

b. You know that $\angle F \cong \angle Q$.

$m\angle F = m\angle Q$

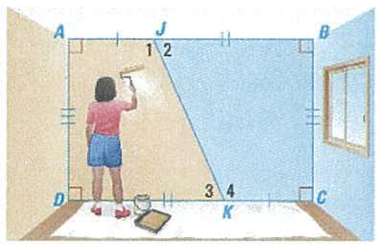
$68° = (6y + x)°$

$68 = 6y + 8$

$10 = y$

EXAMPLE 3 Show that figures are congruent

PAINTING If you divide the wall into orange and blue sections along $\overline{JK}$, will the sections of the wall be the same size and shape? *Explain.*



Solution

From the diagram, $\angle A \cong \angle C$ and $\angle D \cong \angle B$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{AB} \parallel \overline{DC}$. Then, $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent.

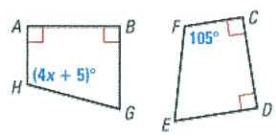
The diagram shows $\overline{AJ} \cong \overline{CK}$, $\overline{KD} \cong \overline{JB}$, and $\overline{DA} \cong \overline{BC}$. By the Reflexive Property, $\overline{JK} \cong \overline{KJ}$. All corresponding parts are congruent, so $AJKD \cong CKJB$.

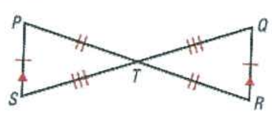
▶ Yes, the two sections will be the same size and shape.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

In the diagram at the right, *ABGH* ≅ *CDEF*.

1. Identify all pairs of congruent corresponding parts.
2. Find the value of x and find $m\angle H$.
3. Show that $\triangle PTS \cong \triangle RTQ$.



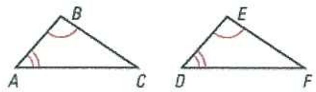


THEOREM — *For Your Notebook*

THEOREM 4.3 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

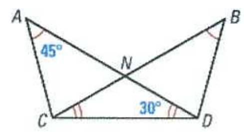
Proof: Ex. 28, p. 230



If $\angle A \cong \angle D$, and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

EXAMPLE 4 Use the Third Angles Theorem

Find $m\angle BDC$.



ANOTHER WAY
For an alternative method for solving the problem in Example 4, turn to page 232 for the **Problem Solving Workshop**.

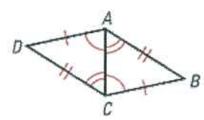
Solution

$\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, so by the Third Angles Theorem, $\angle ACD \cong \angle BDC$. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

▶ So, $m\angle ACD = m\angle BDC = 105°$ by the definition of congruent angles.

EXAMPLE 5 Prove that triangles are congruent

Write a proof.



GIVEN ▶ $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$

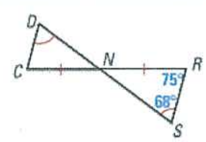
PROVE ▶ $\triangle ACD \cong \triangle CAB$

Plan for Proof
a. Use the Reflexive Property to show that $\overline{AC} \cong \overline{AC}$.
b. Use the Third Angles Theorem to show that $\angle B \cong \angle D$.

Plan in Action

	STATEMENTS	REASONS
	1. $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$	1. Given
a.	2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property of Congruence
	3. $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$	3. Given
b.	4. $\angle B \cong \angle D$	4. Third Angles Theorem
	5. $\triangle ACD \cong \triangle CAB$	5. Definition of $\cong$ △s

✓ GUIDED PRACTICE for Examples 4 and 5



4. In the diagram, what is $m\angle DCN$?
5. By the definition of congruence, what additional information is needed to know that $\triangle NDC \cong \triangle NSR$?

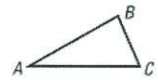
PROPERTIES OF CONGRUENT TRIANGLES The properties of congruence that are true for segments and angles are also true for triangles.

THEOREM *For Your Notebook*

THEOREM 4.4 Properties of Congruent Triangles

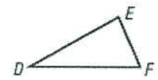
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong \triangle ABC$.



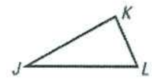
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

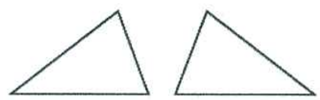


4.2 EXERCISES

HOMEWORK KEY ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 15, and 25
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 21, 24, 27, and 30

SKILL PRACTICE

1. **VOCABULARY** Copy the congruent triangles shown. Then label the vertices of the triangles so that $\triangle JKL \cong \triangle RST$. Identify all pairs of congruent *corresponding angles* and *corresponding sides*.

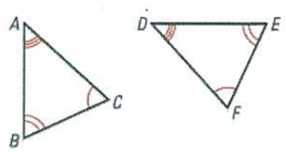


2. ★ **WRITING** Based on this lesson, what information do you need to prove that two triangles are congruent? *Explain.*

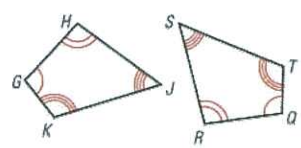
EXAMPLE 1 on p. 225 for Exs. 3–4

USING CONGRUENCE Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures.

3. $\triangle ABC \cong \triangle DEF$



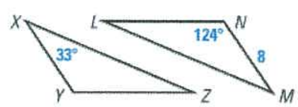
4. $GHJK \cong QRST$



EXAMPLE 2 on p. 226 for Exs. 5–10

READING A DIAGRAM In the diagram, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

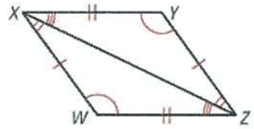
5. $m\angle Y =$ _?_
6. $m\angle M =$ _?_
7. $YX =$ _?_
8. $\overline{YZ} \cong$ _?_
9. $\triangle LNM \cong$ _?_
10. $\triangle YXZ \cong$ _?_



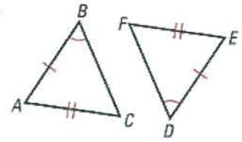
EXAMPLE 3
on p. 226
for Exs. 11–14

NAMING CONGRUENT FIGURES Write a congruence statement for any figures that can be proved congruent. *Explain* your reasoning.

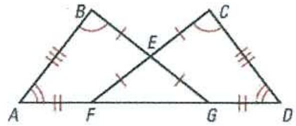
11.



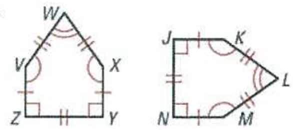
12.



13.



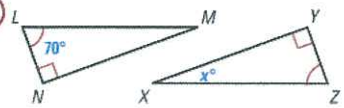
14.



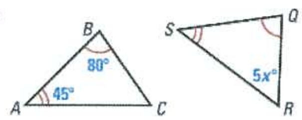
EXAMPLE 4
on p. 227
for Exs. 15–16

THIRD ANGLES THEOREM Find the value of x.

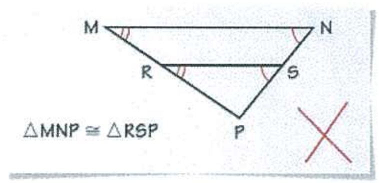
15.



16.



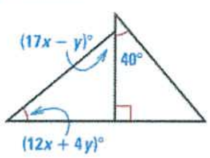
17. ERROR ANALYSIS A student says that $\triangle MNP \cong \triangle RSP$ because the corresponding angles of the triangles are congruent. *Describe* the error in this statement.



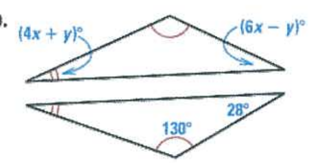
18. ★ OPEN-ENDED MATH Graph the triangle with vertices $L(3, 1)$, $M(8, 1)$, and $N(8, 8)$. Then graph a triangle congruent to $\triangle LMN$.

ALGEBRA Find the values of x and y.

19.



20.



21. ★ MULTIPLE CHOICE Suppose $\triangle ABC \cong \triangle EFD$, $\triangle EFD \cong \triangle GIH$, $m\angle A = 90°$, and $m\angle F = 20°$. What is $m\angle H$?

Ⓐ 20° Ⓑ 70° Ⓒ 90° Ⓓ Cannot be determined

22. CHALLENGE A hexagon is contained in a cube, as shown. Each vertex of the hexagon lies on the midpoint of an edge of the cube. This hexagon is equiangular. *Explain* why it is also regular.